Math 250 **Name (Print):** ____________________

Fall 2013

Quiz 8

1. (10 pts)A matrix and its characteristic polynomial is given. Determine all values of c so that the matrix is not diagonalizable.

$$\begin{bmatrix} 1 & -1 & 0 \\ 6 & 6 & 0 \\ 0 & 0 & c \end{bmatrix}$$
$$-(t-c)(t-3)(t-4)$$

Ans: The only possible choices for c are $c = 3$ and $c = 4$.
When $c = 3$

$$A - cI = \begin{bmatrix} -2 & -1 & 0 \\ 6 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So its simplified form is

$$R = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

which clearly has 2 free variables. Hence when $c = 3$ the matrix IS diagonalizable.
When $c = 4$

$$A - cI = \begin{bmatrix} -3 & -1 & 0 \\ 6 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So its simplified form is

$$R = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

which also clearly has 2 free variables. Hence when $c = 4$ the matrix IS diagonalizable.

Conclusion: There is NO value of c that would make the matrix NOT diagonalizable.

2. (10 pts) A matrix A and its characteristic polynomial is given. Find, if possible, an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

$$\begin{bmatrix} 7 & 6 \\ -1 & 2 \end{bmatrix}$$
$$(t-4)(t-5)$$

Ans: When $t = 4$

$$A - tI = \begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix}$$

so an eigenvector is $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$. When $t = 5$

$$A - tI = \begin{bmatrix} 2 & 6 \\ -1 & -3 \end{bmatrix}$$

so an eigenvector is $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$.

So

$$D = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}$$

$$P = \begin{bmatrix} -2 & -3 \\ 1 & 1 \end{bmatrix}.$$